WEEK 5: Proportion and Scale
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1. Proportion and Scale

Scale and proportion play very important roles for architecture. Proportion refers to the proper and harmonious relation of one part to another or to the whole, while scale refers to the size of something compared to a reference standard or to the size of something else (like a human being).

2. Proportion

The mind seeks out mathematical and geometrical relationships – or proportions – in patterns. Human beings possess a special intuition which makes them perceive simple mathematical proportions in the physical world. This is also true of music. For this reason, since architecture is a composition of forms brought together in proportional relationships, it was called frozen music.

"Proportion" refers to the relative size of visual elements within an image. It also refers to the equality between two ratios in which the first of the four terms divided by the second equals the third divided by the fourth.

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\[ \frac{a}{b} = \frac{c}{d} \]
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Geometry is inevitable in architectural organization as the means of ordering a design and relating the parts to one another. The first proportional relationship begins in architectural design in the material level and in the level of architectural elements. Many architectural elements are sized and proportioned not only according to their structural properties and function, but also by the process through which they are manufactured. Because the elements are mass produced in factories, they have standard sizes and proportions imposed on them by the individual manufacturers or by industry standards.

Concrete block and common brick for example are produced as modular building units. Although they differ from each other in size, both are proportioned on a similar basis. Standard sizes and proportions of factory produced elements affect the size, proportion and spacing of other materials as well. Standard door and window units are also sized and proportioned to fit into modular masonry openings. These dimensions may also determine the dimensions of spaces.

Concrete block and common brick are produced as modular building units
Concrete block and common brick are applied in proportional relationships to each other.

Standard door and window units are sized and proportioned to fit into modular masonry openings.

The designer has still the ability to control the proportion of forms and spaces within and around a building. The functioning of the space and the nature of activities to be accommodated there will influence its form and proportion.

However, the designer can also play with proportions for aesthetic reasons, for producing ‘desirable’ relationships among the dimensions of the parts and the whole of a building. For this reason, various different proportional systems were developed in various different times.

3. **Proportioning systems**
   Throughout history, it has been realized that a proportion system can assist both the ordering and also the perception of buildings. Proportioning systems provide an aesthetic rationale for the dimensions of form and space. They can visually unify the multiplicity of
elements in an architectural design by having all of its parts belong to the same family of proportions. They provide a sense of order in the facades and spaces of architectural works. A number of theories of ‘desirable’ proportions have been developed in the course of history.

Theories of Proportion:
   a. Golden section
   b. Regulating lines
   c. Classical orders
   d. Renaissance theories
   e. Modulor
   f. Ken
   g. Anthrophometry

These proportioning techniques developed were used to shape architecture in different periods and countries.

a. Golden section
The Greeks have found out that nature uses a proportion law called Golden section (and Fibonacci Series), which produces things that look pleasing to us. Golden Section is basically described as the law of beautiful proportions. According to this law, two quantities are said to be in the golden section \((\varphi)\) if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one.

\[
\frac{a + b}{a} = \frac{a}{b} \equiv \varphi,
\]

Here, the Greek letter phi \((\varphi)\) represents the golden section (or ratio). Its value is:

\[
\varphi = \frac{1 + \sqrt{5}}{2} = 1.6180339887\ldots
\]

On this basis, a golden rectangle is:

A rectangle whose sides are proportioned according to Golden Section is called a Golden Rectangle. If a square is drawn in its smaller side, the remaining portion of the rectangle would be a smaller but similar Golden Rectangle. This operation can be repeated indefinitely to produce a gradation of squares and golden rectangles. In this system, each part remains similar to all of the other parts, as well as to the whole.
If a Golden section rectangle is divided by drawing a square in it, the remaining rectangle is again a golden section rectangle. If that remaining rectangle is divided again and this is continued until no more squares could be drawn, in the emerging pattern, the corners of the rectangles could be connected as to form a logarithmic spiral. It was found that the patterns of seeds in plants and also nautilus shells follow this logarithmic spiral.

In mathematics, the successive proportions of a series of numbers, which are called Fibonacci numbers, give the Golden Ratio. In these series, a number is the sum of the two consecutive numbers before itself. If a Fibonacci number is divided by its immediate predecessor in the sequence, the quotient approximates φ (like: 13/8 = φ). The larger the numbers get, the closer it approximates φ. Fibonacci numbers are in the following integer sequence:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

A tiling with squares whose sides are successive Fibonacci numbers in length.
Fibonacci series show themselves in the branching in trees, the arrangement of leaves on a stem, the arrangement of a pine cone etc.

**a.1. Nature and Golden Section**

Nature (the leaves, the trees, the animals, and human beings) develops and grows according to Golden Section (or Fibonacci series).

Plants grow according to golden section

Animals grow according to golden section

Animals grow according to golden section
Universe grows according to golden section

Human beings also are proportioned according to Golden Section. Human hands, arms, ears, teeth, etc. are in phi (golden section) proportions.¹

¹ http://goldennumber.net
Spiral of the Ear, teeth and lips are in Phi proportions (in Golden section)

Phi Proportions (golden section) is used to determine the proportions in cosmetic surgery.

One of the first names who studied human dimensions and proportions according to Golden Section is Leonardo da Vinci. In his famous ‘Vitruvian man’, he drew the ideal man, based on the correlations of ideal human proportions with geometry described as by the
ancient Roman architect Vitruvius. Vitruvius asserted that ideal man’s proportions were based on Golden Section.

![Vitruvian Man by Leonardo da Vinci](image)

Vitruvian Man by Leonardo da Vinci

![Geometric division of length by using Golden Section](image)

Geometric division of length by using Golden Section

**a.2. Arts and Golden Section**

Leonardo Da Vinci used the Golden Section extensively. In Da Vinci’s “The Last Supper” for example, all the key dimensions of the room and the table were based on the Golden section (which was known in the Renaissance period as The Divine Proportion).²

² [http://www.goldennumber.net/art-composition-design/](http://www.goldennumber.net/art-composition-design/)
Da Vinci’s “The Last Supper”; the successive divisions of each section of the painting by the golden section define the key elements of composition.

Golden section was used in arts extensively in different periods. In the “Bathers at Asnières” by Georges Seurat for example, the horizon falls exactly at the golden section of the height of the painting. The trees and people are placed at golden sections of smaller sections of the painting.  

Bathers at Asnières (Une Baignade, Asnières), Georges Seurat, 1884.

In “The Sacrament of the Last Supper”, Salvador Dali framed his painting in a golden rectangle. Following Da Vinci’s lead, Dali positioned the table exactly at the golden section of the height of his painting. He positioned the two disciples at Christ’s side at the golden sections of the width of the composition. In addition, the windows in the background are formed by a large dodecahedron. Dodecahedrons consist of 12 pentagons, which exhibit phi relationships in their proportions.  

3 http://www.goldennumber.net/art-composition-design/
4 http://www.goldennumber.net/art-composition-design/
"The Sacrament of the Last Supper" by Salvador Dali

Golden section was also used in sculptures:
a.3. Architecture and Golden Section

Phi (Φ), the Golden Section, has been used by mankind for centuries in architecture. Its use started as early as with the Egyptians in the design of the pyramids. When the basic phi relationships are used to create a right triangle, it forms the dimensions of the great pyramids of Egypt, with the geometry shown below creating an angle of 51.83 degrees, the cosine of which is phi, or 1.618.  

Source: http://www.goldennumber.net/architecture/

The Greeks used golden section extensively for beauty and balance in the design of the Parthenon and other architectural works⁶:

Parthenon, Athens - Golden section was used in the proportioning of the façade ⁷

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⁵ http://www.goldennumber.net/architecture/
⁶ http://www.goldennumber.net/architecture/
⁷ http://www.goldennumber.net/architecture/
Golden section was used in the design of Notre Dame in Paris, which was built in the 1163 and 1250.  

It was also used extensively by Renaissance artists of the 1500's. It was called as the Divine Proportion in this period.

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8 http://www.goldennumber.net/architecture/
9 http://www.goldennumber.net/architecture/
In India, it was used in the construction of the Taj Mahal, which was completed in 1648.10

Its use continues in modern architecture, as illustrated in the United Nations building11:

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10 http://www.goldennumber.net/architecture/
11 http://www.goldennumber.net/architecture/
The CN Tower in Toronto, the tallest tower and freestanding structure in the world, also contains the golden ratio in its design. The ratio of observation deck at 342 meters to the total height of 553.33 is 0.618 or (the reciprocal of Phi).\(^\text{12}\)

![CN Tower in Toronto](image)

b. **Regulating lines**

The lines that indicate the common alignment of elements are called regulating lines. They are used to control the proportion and placement of elements in architecture. They reassure the perception of order and fix the fundamental geometry of work.

Le Corbusier was a famous supporter of regulating lines and called them as the inevitable element of architecture and the necessity for order. He argued that great architecture of the past has been guided by these regulating lines and these lines, starting at significant areas of the main volumes, could be used to rationalize the placement of features in buildings.

He proposed that the regulating lines of forms (or the geometrical laws of any particular form) should be the basis for subsequent action. Once these geometrical laws are understood and the lines are drawn, various axes can be traced and the properties of forms (whether they are linear, or centroidal, static or dynamic) can be detected.\(^\text{13}\) According to Corbusier, regulating lines guarantee fine proportions and add a rational sense of coherence to the buildings. In this way, the order, the function, and the volume of the space are drawn into one architectural totality. He explained this as follows:

“The regulating line is a guarantee against willfulness. It brings satisfaction to the understanding. The regulating line is a means to an end. It is not a recipe. Its choice and the modalities of expression given to it are an integral part of architectural creation.” (Le Corbusier, Towards New Architecture)

\(^{12}\) http://www.goldennumber.net/architecture/
\(^{13}\) Baker, G., Strategies of Architectural Analysis, p. 45.
In his essay “The mathematics of the ideal villa”, Colin Rowe pointed out the similarity between the spatial subdivision of a Palladian villa and the structural grid of a villa by Le Corbusier (Villa Garches). While both villas shared a similar proportioning system, Palladio’s villa consisted of spaces with fixed shapes and harmonic interrelationships. He used simple ratios of 3:4, 4:4, 4:6, as in musical harmony. The visitor feels these proportions. Not as exact proportions but as the idea behind them. He/she feels that there is an integrated composition, where every part has a meaning in relation to the whole. Le Corbusier’s villa on the other hand was composed of horizontal layers of free space. The rooms varied in shape and were asymmetrically arranged in each level. The geometric net of structural elements is
very similar to Villa Foscarini in terms of their proportions. Corbusier tells that he has used the ratio of 5:8, which is very close to Golden section. However, here, the visitor can not perceive any proportional system that is used in design, since Corbusier consciously suppresses the structural elements. Therefore although the underlining system of proportions is similar, there is no similarity in the principles of composition of these two buildings.

Villa Garches by Le Corbusier
c. Classical order

Classical order is one of the ancient styles of classical architecture, distinguished by its proportions and characteristic details, and mostly by the type of column employed. It is a way of proportioning the elements. Three ancient orders of architecture—the Doric, Ionic, and Corinthian—originated in Greece. To these the Romans added the Tuscan, which they made simpler than Doric, and the Composite, which was more ornamental than the Corinthian.

To Greeks and Romans, the Orders represented the perfect beauty and harmony. The basic unit of dimension was the diameter of the column. From this module, the dimensions of the shaft were derived, as well as the dimensions of the capital, the pedestal below and the entablature above. The spaces between the columns were also based on the diameter of the column. In this system, the intention was to ensure that all of the parts of any one building were proportionate and in harmony with one another.

The Parthenon is a temple of the Doric order with eight columns at the façade, and seventeen columns at the flanks, conforming to the established ratio of 9:4. This ratio governed the vertical and horizontal proportions of the temple as well as many other relationships of the building like the spacing between the columns and their height.
Parthenon, in Athens, Greece, 432 BC
d. Renaissance theories

The architects of the Renaissance, believing that their buildings had to belong to a higher order, returned to the Greek mathematical system of proportions. Just as Greeks thought music to be the geometry translated in sound, the Renaissance architects thought that architecture was mathematics translated into spatial units. They applied the proportioning system developed by Pythagoras (which was based on musical scale), and developed a progression of ratios that formed the basis of their architecture. These proportions manifested
themselves not only in the dimensions of a room or façade, but also in the relationship of spaces in the plan.

Andrea Palladio (1508-1580) was probably the most influential architect of the Italian Renaissance. He was influenced by Roman and Greek architecture, primarily by Vitruvius. He used classical principles to design a series of country villas and urban palaces for the nobility of Vicenza. He freely recombined elements of Roman buildings in his own building sites according to contemporary needs (creating houses with temple fronts). At the same time he shared the Renaissance concern for harmonious proportion. He thought that beauty will arise from the relationships of the parts to each other and to the whole. His facades have a noteworthy simplicity arising from the use of proportions. One of his most famous is Villa Capra, also known as the Rotunda, which was modeled after the Roman Pantheon. Palladio influenced British American and European architecture in the past four hundred and fifty years, mainly because of his book *Four Books on Architecture*. 
Villa Capra (or Villa Rotunda) by Palladio, Vicenza, Italy (1552-67)

Villa Barbaro by Palladio

e. **Le Modulor**

The famous architect Le Corbusier also worked with human proportions and Golden Section. He has developed a theory of proportion and dimensioning system, named *Modulor* that is based on Golden Section and human proportions. He had formed the proportions of human body according to Fibonacci series and accepted the average human height as 183 cm (He found out that height also according to Fibonacci Series).

According to that, he had developed two series of dimensions, first according to the full height of the man, which is 183 cm (red series) and the second according to the height when he lifted his arm, which is 226 cm (blue series). He developed this system for the serial production of standard furnitures and for determining the lengths, heights and widths of inner spaces. He believed that Le Modulor satisfied both the demands of beauty (because it is derived from Golden section/Fibonacci series) and also functional demands.

Red series: 4-6-10-16-27-43-70-113-183 cm.

These proportional dimensions were used for detecting the following dimensions:
27 cm. - the height of the armchair while sitting
43 cm - the height of the chair
70 cm - the height of the table
86 cm - the height of the countertop
113 cm - the height of the bar
140 cm - the height of the armchair horizontal arm
183 cm - human height
226 cm - human height with arm lifted up
Le Corbusier’s Le Modulor

Le Corbusier used his Modulor dimensions in the design of Unite d’Habitation Residential Block in Marseilles, France. He used 15 measures of Modulor to bring human scale to the huge building, which is 140 m’s long, 24 m’s wide and 70 m’s high.

Le Corbusier’s Unite d’Habitation in Marseilles, France

Section, Le Corbusier’s Unite d’Habitation in Marseilles, France
Modulor carvings in the facades, Le Corbusier’s Unite d’Habitation in Marseilles, France

f. Ken

Ken is the Japanese unit of measure. It originally designated the interval between two columns and it was standardized later for residential architecture. Ken was used as the absolute measurement for the construction of buildings and as an aesthetic module that ordered the structure, materials and space of Japanese architecture.

Together with ken, another module also affected the design of Japanese spaces, which was the tatami (or the traditional Japanese floor mat). Tatami was originally proportioned to accommodate two persons sitting or one person sleeping. The smaller side of a tatami was equal to the size(s) of ken and two kens equaled the length of a tatami. The Japanese inner spaces were formed according to ken modules and the placement of tatamis. The size of a room was designated by the number of tatamis.

Traditional Japanese house and the use of ken modules
Traditional Japanese house and the use of ken modules

In a traditional Japanese house the ken grid orders the structure as well as the additive sequence of rooms.
Anthropometry refers to the size and proportions of the human body. Anthropometric proportioning methods search for the functional dimensions for the human body. They say that forms and spaces in architecture are either containers or extensions of the human body and therefore they should be designed according to its dimensions.

The sizes and shapes of building elements and components are the outcomes of human dimensions and his/her certain functions. Every furniture that man created are produced according to his/her dimensions. For this reason, in the past, dimensions of human organs (such as the finger, foot, arm etc.) were used as the standard dimension units:

**British:**
- Inch (One finger): 2.54 cm.
- Foot (One foot): 30.48 cm.
- Yard (Three feet): 91.44 cm.

**Turkish:**
- Endaze (One open arm): 65.25 cm.
- Arşın (One open arm+1/4 chest): 68.58

Accordingly, the dimensions and dimensions systems we use today have been originated from human proportions and dimensions. Human being is the basis of all dimensions. Human dimensions and the relations between those dimensions (proportions) have been studied for a very long time. Anthropometrics deals with this subject and detects those dimensions, because the dimensions and proportions of the human body affect the proportion of the things we handle, the dimensions of our furniture, and the dimensions of the spaces we use. (For this reason, in order to coordinate the design of devices, systems, and
environments with our physiological and psychological capacities and requirements, another branch of science emerged, which is called ergonomics.)

The fit between form and dimensions of a space and our own body dimensions can be a static one as when we sit in a chair, or lean against a wall, and can be a dynamic one as when we walk up a stairway or move through the rooms of buildings. An architect should plan and design his buildings in accordance with human dimensions in an ergonomic manner. He/she should design the furnitures according to human dimensions and design spaces according to those furnitures.

Buildings scaled to human dimensions and physical capabilities in an ergonomic manner have architectural elements, such as steps, doorways, railings, work surfaces, seating, shelves, fixtures etc. that fit well to the average person. Moreover, the spaces that are left between these elements should be arranged carefully so that a human being can act and move efficiently between them. The dimensions of these spaces and the circulation areas are determined according to the dimensions of furniture and the standard dimensions of a human being in static and dynamic positions.
Spaces could be formed by way of taking human dimensions and the furniture he/she uses as the basis. For example, the plan of Frederik's Hospital in Copenhagen, designed by Danish architect Nicolai Eigtvet, was formed by taking the hospital bed as the module, which was placed in 180 cm apart from the other beds. In this building the dimensions were not determined by golden section or modulor, but by the beds which the hospital was built to hold.
While proportion refers to an ordered set of mathematical relationships among the dimensions of a form and space, scale refers to how we perceive the size of something in relation to something else. It is a proportional relationship between two sets of dimensions. In dealing with the issue of scale therefore, we are always comparing one thing to another.


Scale also refers to the mathematical relationship between an object and a measurable quantity (the scale referent). In an architectural drawing, we use a scale to specify the ratio that determines the relationship between the drawing and the actual building. We say that an object is “full-scale” when it corresponds 1 to 1 (1:1) with real life. If the same object is rendered such that any given linear dimension is one-half the length of the original object, we understand this to be at “half scale” or 1:2.

a. Visual scale

Psychologists have studied how we perceive visual information and their findings have showed that we perceive and judge the size of something in relation to something else. A thing appears smaller or larger in relation to the size of other things in its environment.
One of the studies is known as the Ebbinghaus Illusion. The Ebbinghaus illusion (or Titchener circles) is the optical illusion that shows relative size perception. In this illusion, two circles of the same size are perceived to be different sizes because of the size of the surrounding objects.¹⁴

Another example is the Muller-Lyer Illusion where line segments of the same size appear to be of different lengths based on the shapes that are placed at the ends.

We perceive the size of a building in comparison to the other structures near it (or human beings near it) or we perceive the size of elements (such as doors or windows in a building) in relation to the other parts or to the whole of a composition. We also understand the true sizes of the buildings when we compare them to standard elements such as residential window units or modular materials such as bricks.

We understand the size of elements in relation to the other parts or to the whole of a composition

CN Tower – height is understood by comparison to the other structures near it. (Source: http://zonfiza.blogspot.com/2010/11/proportion-and-scale.html)

Photo realistic giant sculptures of Ron Mueck play with the scale of human figures

The sculptures of Claes Oldenburg play with the scale of every day objects
The sculptures of Claes Oldenburg play with the scale of every day objects

b. Human scale

Human scale refers to the size of a form when compared with our own human size. Human dimensions and scale have a determinative effect both in our perception and also in our creation of buildings and spaces. Human scale in architecture is based on the dimensions and proportions of the human body.

Architecture is the largest of the visual arts. In terms of perception, one of the challenges for the user is to determine just how big a building is. For doing this we use our own size as the reference. We can understand the dimensions of a building better, when we compare that to our human dimensions. How big a building is relative to the size of the average human being is said to be its scale.

We can understand the true dimensions of the building better when we compare it to human beings.
We can understand the true dimensions of the building better when we compare it to human beings.

While something monumental in scale makes us feel small in comparison, a space that is intimate in scale describes an environment. The difference between monumental scale and domestic scale results from the difference of proportions. In monumental architecture the harmony is created by the proportioning of ultra large elements, where as in domestic (human scale) architecture, harmony is created by the proportioning of human scale elements (doors, windows etc.).

Size an scale in architecture: Monumental Architecture-grand in size - Sacre Coeur Basilica, Paris, France (left); Domestic Architecture-humble in size - Mudbrick houses in Yaprakbası village, Turkey (right)

We can understand the size of a space, when we compare it to the elements whose dimensions are related with our dimensions, such as windows, doors etc.
One example where this kind of perception, which is done by comparing the building to its elements (for understanding its dimensions), is consciously distorted is Saint Peter’s Basilica in Rome by Michelangelo. Michelangelo consciously enlarged the sizes of windows and doors by two or three times than their normal sizes to be able to give this huge building a simplistic composition.

In modern architecture of 20th century, architects again chose to ignore the clues of size arising by common building elements. For example Beinecke Rare Book Library in Yale University, provide very few number of clues as to its size relative to human beings. The older buildings in the surrounding context on the other hand, however provide many clues about it. Only when there are people around the true size of Beinecke Rare Book Library is understood.
Beinecke Rare Book Library in Yale University